Universidad Industrial de Santander

High Dimensional Signal Processing Research Group

**Grupo de Investigación HDSP**
Escuela de Ingeniería de Sistemas e Informática, Universidad Industrial de Santander

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# Taller de Laboratorio No. 1
# Formación de Imágenes y Adquisición Espectral

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## Workshop overview

**Pipeline:** component identification → image formation → spectral GT (31 bands) → filter measurements ($N \approx 20$) → reconstruction → metrics.

**Output:** report (2–4 pages) with figures, a small metrics table, and discussion.

## Checklist

- A personal scene setup.
- Identification and description of the experimental components.
- Evidence of image formation and optical setup alignment.
- Ground truth hyperspectral cube (31 bands) acquired with the monochromator.
- Multispectral acquisition with a filter wheel ($N \approx 20$ filters), including a capture log (exposure/gain per band).
- Spectral reconstruction results: at least one GT vs reconstructed spectrum plot and a small metrics table.
- A short paragraph on *decisions and fixes*: what was changed and why (alignment, normalization, exposure choices, etc.).

# 1 Theory

Modern imaging systems are present in smartphones, scientific cameras, satellites, medical devices, and industrial inspection systems. Although these systems may appear very different, they all follow the same basic principle: light coming from a scene is collected by an optical system, focused onto a sensor, and converted into digital information that can later be processed and analyzed. Understanding this process requires knowledge from both optics and imaging. First, we must understand how a lens forms an image and how parameters such as focal length and aperture affect image quality. Then, we can study how color and spectral information are captured by imaging sensors. This laboratory introduces these concepts and progressively extends them from conventional RGB imaging to multispectral and hyperspectral acquisition.

## 1.1 Basic Properties of Lenses

A lens does two essential things: it focuses light and controls how much light enters the system. The **focal length** $f$ measures the strength of the lens. A simple way to understand it is this: if parallel rays arrive at the lens (like light coming from a very far object), the lens bends those rays and they meet at one point. That point is located at a distance $f$ from the lens, and this distance is called the focal length.

The **aperture** (with diameter $D$) is like a window that lets light pass through the lens. If the window is large, more light enters; if the window is small, less light enters. The **f-number** or $f/\# = \frac{f}{D}$ is a compact way to describe this relation. It helps us understand two important effects: (1) how bright the image will be on the sensor, and (2) how much of the scene appears sharp (depth of field).

Finally, the **optical power** $P = \frac{1}{f}$ (with $f$ in meters) is often used in optics and ophthalmology. It is expressed in diopters. Table 1 summarizes these basic ideas.

## 1.2 Thin Lens Equation and Image Size

The focus position depends on the object location. If the object is at a distance $d_o$ from the lens and the image forms at a distance $d_i$, both distances are related by the **thin lens equation**:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}. \tag{1}$$

This equation is very useful in practice. For example, if we know the focal length $f$ and we choose where to place the object ($d_o$), the equation tells us where the image plane will be ($d_i$). This is exactly what we need when we build an optical setup on a rail: we must place the sensor at the correct position to get a sharp image.

**Table 1:** Lens–image system properties

| **Quantity** | Simple explanation | Symbol | Useful formula |
|---|---|---|---|
| **Focal length** | Where the lens focuses rays coming from far away. | $f$ | — |
| **Aperture** | Size of the opening that lets light pass. | $D$ | — |
| **F-number** | Controls brightness and depth of field. | $f/\#$ | $\frac{f}{D}$ |
| **Power** | Lens strength in diopters ($\mathrm{m}^{-1}$). | $P$ | $\frac{1}{f}$ |

The **image size** is obtained from similar triangles. If the object has size $y_o$ and the image has size $y_i$, then:

$$y_i = \frac{d_i}{d_o}\, y_o, \tag{2}$$

and the **magnification** $M$ is:

$$M = \frac{d_i}{d_o} = \frac{y_i}{y_o}. \tag{3}$$

Magnification tells us how the size changes. If $M > 1$, the image is larger than the object. If $M < 1$, the image is smaller. In the lab, a very practical configuration is the $2f$ setup. If we place the object at approximately $2f$, then the image forms at approximately $2f$ and the size is similar ($M \approx 1$). This makes focusing easier and gives a stable starting point.

Figure 3 shows the basic idea of image formation with a thin lens.

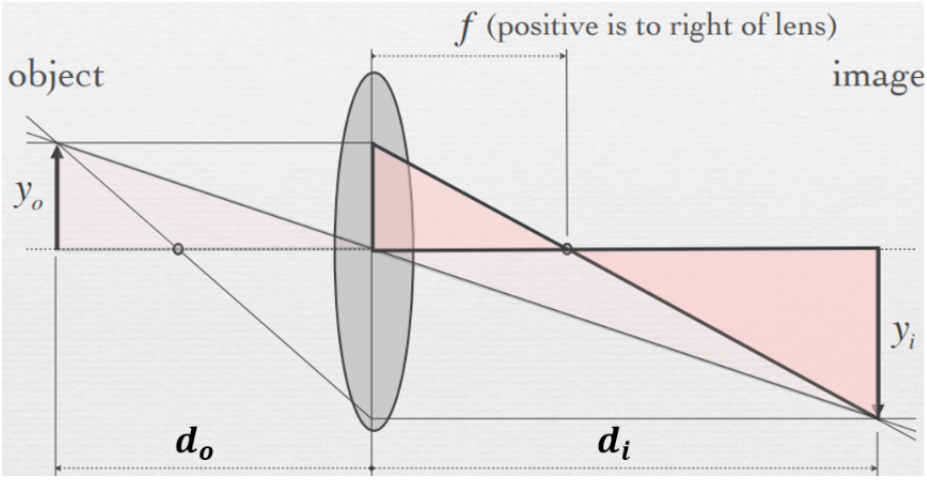



**Figure 1:** Image formation using a thin lens and a practical $2f$ configuration.

**Table 2:** Frequently used formulas

| **Concept** | Expression |
|---|---|
| **Thin lens equation** | $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$ |
| **Image size** | $y_i = \frac{d_i\, y_o}{d_o}$ |
| **Magnification (scale)** | $M = \frac{d_i}{d_o} = \frac{y_i}{y_o}$ |

## 1.3 F-number and Exposure

The f-number $f/\# = \frac{f}{D}$ behaves like a light knob. If we decrease $f/\#$ (larger aperture), more light reaches the sensor and the image becomes brighter. If we increase $f/\#$ (smaller aperture), less light reaches the sensor and the image becomes darker. At the same time, the f-number also affects depth of field: a larger aperture usually produces more blur outside the focus plane, while a smaller aperture increases the sharp region.

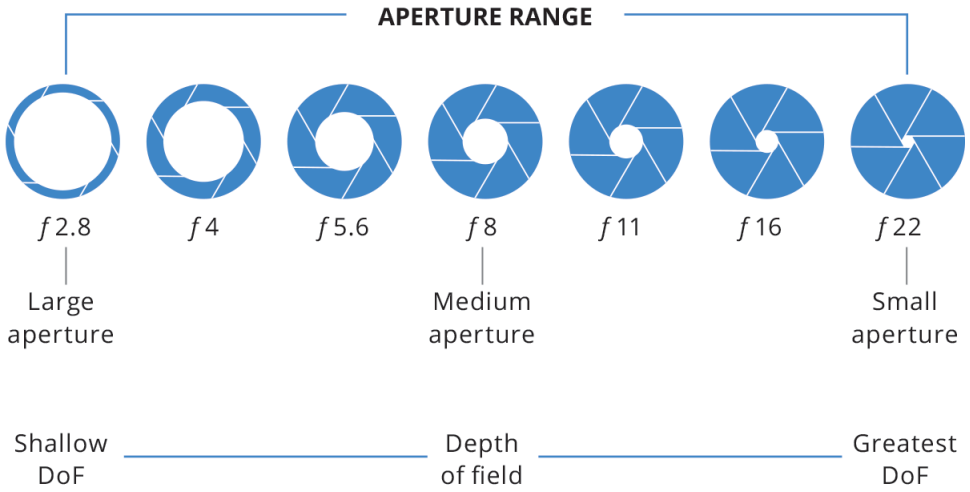



**Figure 2:** Effect of the f-number on aperture size. Lower f-numbers correspond to larger openings and more light, while higher f-numbers correspond to smaller openings and less light.

A simple rule is that, for the same scene and exposure time, the brightness at the sensor changes approximately as:

$$\frac{1}{(f/\#)^2}. \tag{4}$$

A common example: going from $f/4$ to $f/8$ reduces the light to one quarter.

## 1.4 Dioptric Power

When we measure $f$ in meters, the **power** $\Phi = 1/f$ is expressed in diopters. This is useful when we combine thin lenses in contact: as a first approximation, their powers can be added.

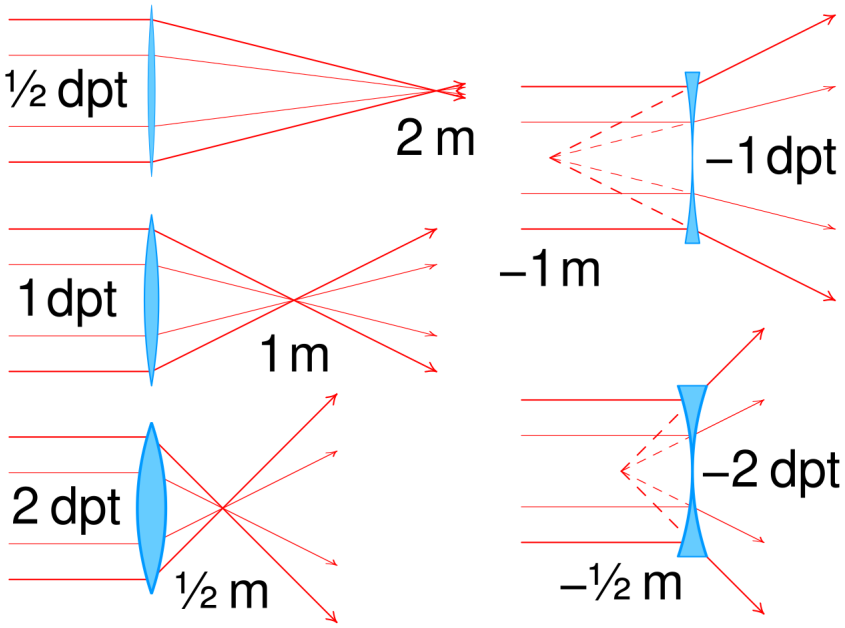



**Figure 3:** Relation between focal length and optical power. Optical power is defined as the inverse of focal length in meters; therefore, shorter focal lengths correspond to stronger lenses.

$$\Phi = \frac{1}{f}. \tag{5}$$

## 1.5 From RGB to Grayscale

So far, we have discussed how an optical system forms an image on a sensor. However, image formation alone does not explain how color information is represented digitally. A camera sensor records light intensity, but many applications require color information to distinguish materials, objects, and scene content. Therefore, it is necessary to understand how color information can be represented, processed, and eventually extended toward spectral imaging. A grayscale image assigns one number to each pixel. If the original pixel is $(R, G, B)$, there are several simple ways to obtain a gray value $Y$. Each method produces a slightly different visual result. In a conventional RGB camera, color is usually obtained using red, green, and blue channels. However, the same idea can be extended using more filters: instead of only three broad color channels, we can acquire several filtered images at different spectral regions. This is the first step toward multispectral and hyperspectral imaging.

The most direct method is the **average**:

$$Y = \frac{R + G + B}{3}. \tag{6}$$

Another method is the **lightness (clarity)** approach, using the maximum and minimum channel values:

$$Y = \frac{\max(R, G, B) + \min(R, G, B)}{2}. \tag{7}$$

A more realistic method is **luminance** (perceptual), which gives more weight to the green channel because the human eye is more sensitive in that region. For sRGB, typical weights are $w_R \approx 0.2126$, $w_G \approx 0.7152$, and $w_B \approx 0.0722$:

$$Y = w_R R + w_G G + w_B B. \tag{8}$$

Figure 4 compares the three methods.



(a) Imagen original (b) Claridad (c) Promediado (d) Luminosidad

**Figure 4:** Three ways to convert RGB to grayscale: lightness (max–min), average, and luminance (perceptual).

## 1.6 How Color is Captured: Bayer CFA and Demosaicking

Most cameras use a single sensor with a **Color Filter Array (CFA)** on top. The most common CFA is the Bayer pattern (RGGB). It includes twice as many green pixels as red or blue, because green contributes strongly to brightness detail (luminance) and helps improve signal-to-noise ratio.

Since each pixel measures only one color, the camera must reconstruct the missing two channels. This process is called **demosaicking**. A simple baseline approach is bilinear interpolation. As an intuition, it can be seen as a filtering step using simple kernels. During bilinear demosaicking, the missing color values are estimated from neighboring pixels. This operation can be interpreted as a convolution process, where local pixel values are combined using interpolation kernels. The following matrices illustrate simplified interpolation filters commonly associated with bilinear reconstruction of red, blue, and green channels.

$$F_{R,B} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}, \tag{9}$$

$$F_G = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 0 \end{bmatrix}. \tag{10}$$

These kernels assign larger weights to nearby pixels and smaller weights to distant pixels. Although modern cameras use more sophisticated demosaicking algorithms, bilinear interpolation provides an intuitive example of how missing color information can be reconstructed from neighboring measurements. Figure 5 illustrates the Bayer sampling and the mosaic idea.

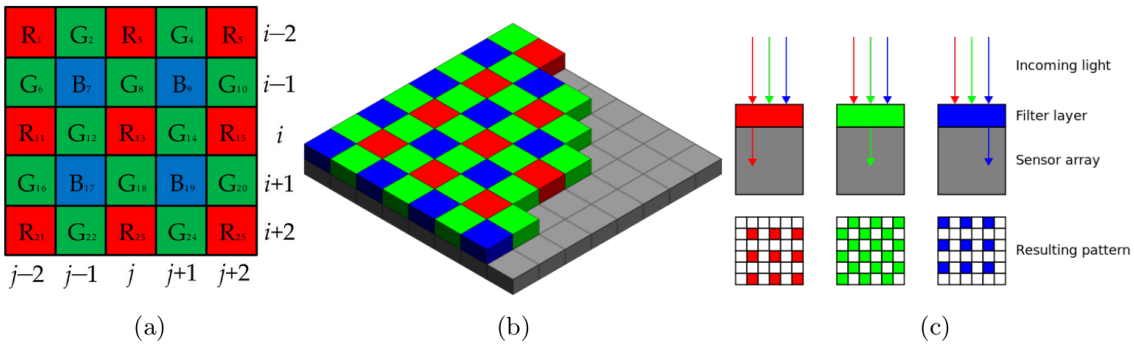



**Figure 5:** Bayer CFA and sampling mosaic: the sensor measures one color per pixel, and demosaicking reconstructs the other two.

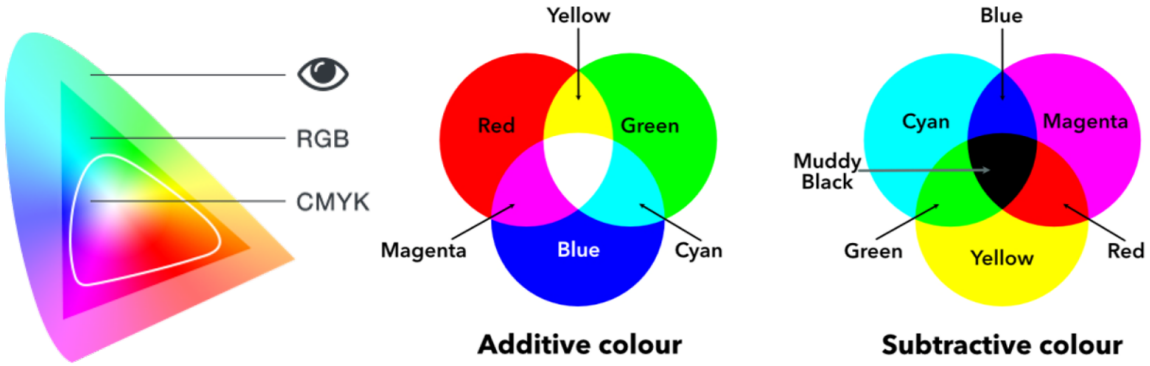
Once the sensor measurements have been reconstructed through demosaicking, the resulting image contains numerical RGB values for every pixel. However, these values alone do not completely define a color. Their interpretation depends on the color representation framework being used. This leads naturally to the concepts of color models and color spaces.

## 1.7 Color Models and Color Spaces

A **color model** is a mathematical way to describe colors using numbers (for example *RGB* or *CMYK*). A **color space** defines the real meaning of those numbers so that colors can be consistent across devices. A color space specifies primaries, a white point, and transfer curves (for example *sRGB*, *AdobeRGB*, or *Rec.709*).

In **additive synthesis** (RGB), colors are made by adding light. Black means no light, and white is the maximum combination of $R$, $G$, and $B$. In **subtractive synthesis** (CMYK), colors are made by removing (absorbing) light with inks on white paper. The extra black channel $K$ helps create deeper blacks and controls ink usage.

A useful way to imagine RGB is as a 3D space: the axes are $R$, $G$, and $B$, and the diagonal $R{=}G{=}B$ contains all grayscale values. Figure 6 summarizes these ideas.




(a) Gamut RGB vs CMYK (b) Colores Aditivos y Substractivos

**Figure 6:** Color spaces and color synthesis: additive (RGB) and subtractive (CMYK).

These color spaces play a fundamental role in digital imaging because they define how colors are represented, stored, and displayed across different devices. In practical applications, the choice of color space affects visualization, image processing, printing, and quantitative analysis. Understanding these concepts is important before extending image acquisition beyond three color channels into multispectral and hyperspectral representations.

At this point, we already have a useful map: the thin lens equation tells us *where* the image forms, and magnification $M$ tells us *how large* it will be. The f-number $f/\#$ controls *how much* light reaches the sensor. For color, we understand how a real scene is captured by a sensor with filters (CFA) and how we can recover a full RGB image.

## 1.8 Image Dependence on Wavelength

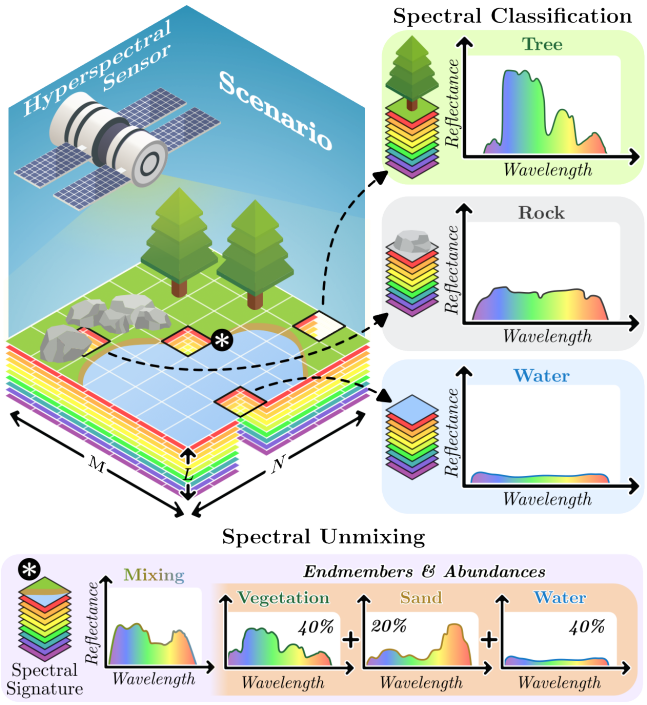



**Figure 7:** Spectral information: the same scene can have different values at different wavelengths.

Until now, we described image formation in a geometric way and color with RGB. However, a real scene does not emit or reflect only three values. In general, it has a continuous spectral distribution. This means the intensity recorded by an optical system depends on wavelength $\lambda$.

For each point $(x, y)$, the recorded signal can be modeled as:

$$I(x,y) = \int_{\lambda_{\min}}^{\lambda_{\max}} S(x,y,\lambda)\, E(\lambda)\, Q(\lambda)\, d\lambda, \tag{11}$$

where $S(x, y, \lambda)$ is the spectral reflectance of the object, $E(\lambda)$ is the spectral distribution of the illuminant, and $Q(\lambda)$ is the spectral response of the optical system and sensor.

This model tells us that the final image is not only a function of space $I(x, y)$, but the result of integrating spectral information weighted by the acquisition system. In RGB cameras, this integration is done through three sensitivity functions (R, G, and B). If we want more spectral detail, we need to acquire more bands.

## 1.9 Spectral Information and Hyperspectral Cube

Instead of integrating over wavelength, we can measure the scene in discrete spectral intervals. Each interval is centered at a wavelength $\lambda_k$, and it produces one image:

$$I(x, y, \lambda_k), \qquad k = 1, \ldots, L. \tag{12}$$

If we stack these images, we obtain a **hyperspectral cube**. Two dimensions correspond to space $(x, y)$, and the third dimension corresponds to wavelength $\lambda$. For each pixel, we can define a spectral vector:

$$\mathbf{s}(x, y) = \begin{bmatrix} I(x, y, \lambda_1) \\ I(x, y, \lambda_2) \\ \vdots \\ I(x, y, \lambda_L) \end{bmatrix} \in \mathbb{R}^L. \tag{13}$$

In a standard RGB camera, $L = 3$. In multispectral imaging, $L$ can be around 5–20 bands. In hyperspectral imaging, the number of bands is often larger and the bands are usually closer together.

In this laboratory, the monochromator will provide a set of $L = 31$ spectral bands that we will use as a reference (*ground truth*). On the other side, the discrete filters will provide fewer measurements $N$, typically $N \approx 20$. The goal is to study how well these $N$ measurements can reconstruct the full 31-band spectrum.

## 1.10 Spectral Acquisition Methods

Once spectral information is represented as a hyperspectral cube, a practical question arises: how can such a large amount of information be physically acquired? Conventional image sensors are inherently two-dimensional devices, whereas hyperspectral data contain two spatial dimensions and one spectral dimension. As a result, additional acquisition strategies are required to sample both spatial and spectral information.

Different spectral imaging architectures have been developed to solve this problem. These architectures differ in how they distribute the acquisition process across space, wavelength, and time. The most common approaches are whiskbroom, pushbroom, and snapshot acquisition.

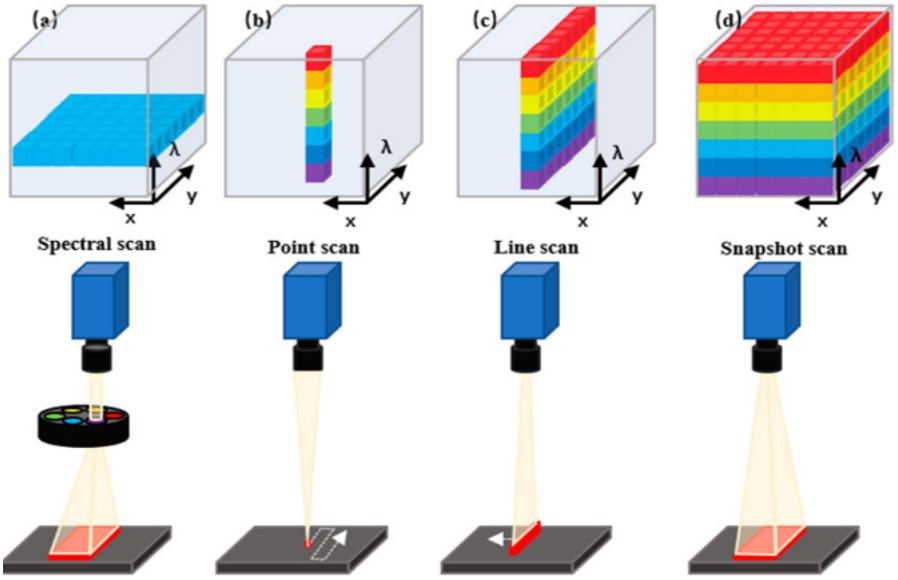



**Figure 8:** Spectral acquisition methods: whiskbroom (point scan), pushbroom (line scan), snapshot (single-shot), and discrete filtering (multiple captures with different filters).

Spectral acquisition systems can be classified by how they separate spatial information and spectral information. In practice, there are three main architectures: **whiskbroom**, **pushbroom**, and **snapshot**.

In **whiskbroom** systems, the scene is scanned point by point using a dispersive element (for example a prism or diffraction grating) and a single detector. Spatial and spectral information are collected sequentially, which can be very accurate but slow.

In **pushbroom** systems, the sensor captures one full spatial line for all wavelengths at the same time. The second spatial dimension is obtained by moving the sensor or the scene. This is common in airborne and satellite systems.

In **snapshot** systems, spatial and spectral information are captured in one exposure. This can be done using filter arrays, beam splitters, or spectral coding. Snapshot systems are fast, but they often have trade-offs in spectral resolution or optical complexity.

Besides these architectures, a simpler approach is **discrete spectral filtering**. In this approach, we use filters with transmission functions $T_k(\lambda)$ placed in front of the sensor. Each capture integrates the scene weighted by the filter response. If we have $N$ different filters, we obtain $N$ different measurements for each pixel.

In this laboratory, we use two complementary approaches. The monochromator will work as a reference system, providing $L = 31$ spectral bands in a controlled way. The filter wheel will provide approximately $N \approx 20$ discrete measurements. These filter measurements are not the full spectrum, but projections of the spectrum through the filter transmission curves. The next step is to model this process mathematically.

## 1.11 Linear Spectral Acquisition Model

To describe the measurement process, we represent the true spectrum at a pixel as a discrete vector:

$$\mathbf{s}(x, y) = \begin{bmatrix} s(\lambda_1) \\ s(\lambda_2) \\ \vdots \\ s(\lambda_L) \end{bmatrix} \in \mathbb{R}^L, \tag{14}$$

where $L$ is the total number of spectral bands (here $L = 31$, from the monochromator).

When we use a spectral filter with transmission $T_k(\lambda)$, the sensor measurement is a weighted combination of the spectrum:

$$i_k = \int s(\lambda)\, T_k(\lambda)\, d\lambda. \tag{15}$$

If we discretize wavelength, the integral becomes a sum:

$$i_k = \sum_{\ell=1}^{L} A_{k\ell}\, s(\lambda_\ell), \tag{16}$$

where the coefficients $A_{k\ell}$ include the filter transmission, the sensor spectral response, and possibly the illuminant spectrum.

If we have $N$ filters, the measurements can be written as a vector:

$$\mathbf{i} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} \in \mathbb{R}^N. \tag{17}$$

The full system can be written in matrix form $\mathbf{i} = \mathbf{A}\mathbf{s}$, where $\mathbf{A} \in \mathbb{R}^{N \times L}$ is the spectral acquisition matrix.

This model shows that each filter measurement is not a "pure" spectral band. Instead, it is a linear projection of the complete spectrum. In this lab, the monochromator gives the full spectrum $\mathbf{s} \in \mathbb{R}^{31}$, while the filter wheel gives a reduced measurement vector $\mathbf{i} \in \mathbb{R}^N$ with $N \approx 20$.

Because $N < L$, recovering $\mathbf{s}$ from $\mathbf{i}$ is an inverse problem and it can be underdetermined. For this reason, we often need additional assumptions or regularization.

## 1.12 Spectral Reconstruction as an Inverse Problem

The acquisition model can be written as $\mathbf{i} = \mathbf{A}\mathbf{s}$, where $\mathbf{s} \in \mathbb{R}^L$ is the true spectrum (31 bands) and $\mathbf{i} \in \mathbb{R}^N$ are the measurements from $N \approx 20$ filters. Since $N < L$, the system is underdetermined: many different spectra $\mathbf{s}$ can produce the same measurement $\mathbf{i}$. Therefore, spectral reconstruction needs extra assumptions that reduce the solution space. A basic solution is the Moore–Penrose pseudoinverse:

$$\hat{\mathbf{s}} = \mathbf{A}^{+}\mathbf{i}, \tag{18}$$

which gives a minimum-norm least-squares solution. However, if $\mathbf{A}$ is ill-conditioned or the problem is strongly underdetermined, this solution may amplify measurement noise.

To stabilize the reconstruction, we can use regularization. A simple method is Tikhonov regularization:

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \|\mathbf{i} - \mathbf{A}\mathbf{s}\|_2^2 + \alpha \|\mathbf{L}\mathbf{s}\|_2^2, \tag{19}$$

where $\alpha > 0$ is the regularization parameter and $\mathbf{L}$ can be chosen as the identity matrix or as a discrete derivative operator that enforces spectral smoothness. The closed-form solution is:

$$\hat{\mathbf{s}} = \left(\mathbf{A}^T\mathbf{A} + \alpha\mathbf{L}^T\mathbf{L}\right)^{-1}\mathbf{A}^T\mathbf{i}. \tag{20}$$

Another physically reasonable assumption is that many natural spectra live in a low-dimensional subspace. In that case, we can write:

$$\mathbf{s} \approx \mathbf{B}\mathbf{c}, \tag{21}$$

where $\mathbf{B}$ is a reduced spectral basis and $\mathbf{c}$ is a coefficient vector with smaller dimension. This reduces the number of unknowns and can improve the stability of the inverse problem.

In this laboratory, spectral reconstruction means estimating a spectrum $\hat{\mathbf{s}}$ from the filter measurements and comparing it with the ground truth spectrum obtained with the monochromator. The reconstruction quality can be evaluated using metrics such as mean squared error per band or average spectral error.

# 2 Objectives

**Learning objectives**

1. Implement image formation on an optical rail and validate focus using the thin lens model.
2. Assemble and align a basic image formation system using a test scene, lens, and monochrome camera.
3. Acquire a hyperspectral ground truth cube using the monochromator ($L = 31$ spectral bands).
4. Acquire multispectral measurements using a filter wheel under broadband illumination ($N \approx 20$ filters).
5. Model the acquisition as a linear system and reconstruct spectra using a basic inverse method.
6. Evaluate reconstruction quality using plots (spectra, selected bands) and quantitative metrics.

# 3 Laboratory Implementation

**Note: Experimental workflow**

**Mission:** Build a simple spectral imaging setup and acquire two complementary datasets: a 31-band reference cube and a reduced set of filter-based measurements.
**Deliverable:** Experimental evidence of the optical setup, monochromator acquisition, filter acquisition, and a short description of the data preparation steps.

## 3.1 Experimental Components

The experiment uses a compact optical setup assembled on an optical breadboard. The main components include a monochrome camera, optical mounts, an imaging lens, a monochromator, discrete spectral filters, a broadband lamp, and a printed test scene. The test scene should contain visible edges, colors, and high-contrast regions, because these details help during focusing, alignment, and later image registration.

Before mounting the system, the components are organized and identified. This step is important because it allows the student to understand the role of each element in the imaging chain: the scene provides the spatial and spectral content, the lens forms the image, the

camera records the intensity, the monochromator provides spectral reference illumination, and the filters generate reduced spectral measurements.




**Figure 9:** Main components used in the experiment, including the monochrome camera, optical mounts, imaging lens, spectral filters, illumination elements, and the printed test scene.

**Note: Image formation setup**

**Mission:** Assemble the basic image formation system using the test scene, the lens, and the monochrome camera.
**Deliverable:** One figure showing the aligned optical path and a short explanation of how focus was achieved.

## 3.2 Image Formation Setup

The first experimental step is to form a sharp image of the scene on the monochrome camera. The object, lens, and camera are placed approximately along the same optical axis. The lens position and camera position are adjusted until the printed scene appears clearly focused on the sensor.

This stage connects directly with the thin lens model discussed in the theory section. In practice, the exact position of the camera is adjusted manually because real mounts, lens thickness, and mechanical constraints introduce small differences with respect to the ideal thin-lens equation. The goal is not to obtain a perfect theoretical distance, but to obtain a stable and repeatable focused image.

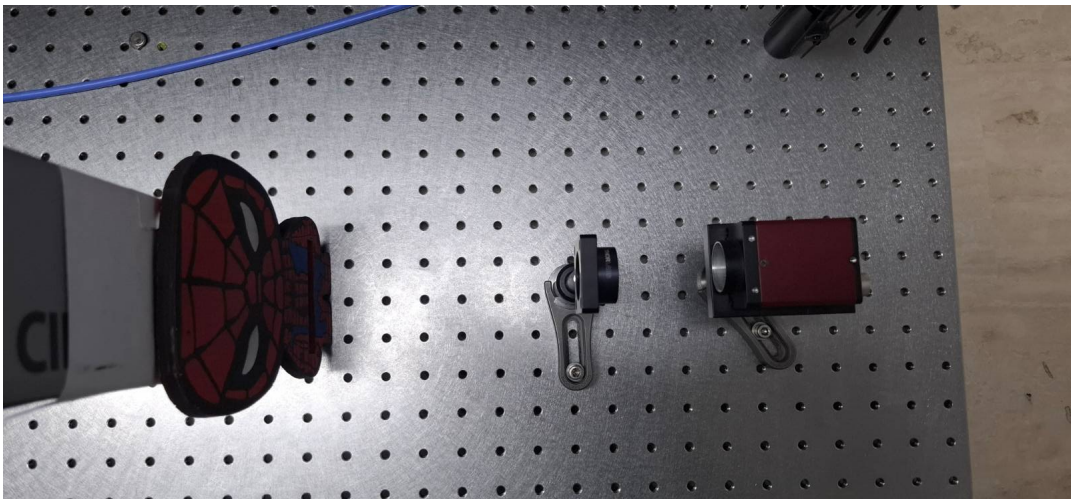


**Figure 10:** Basic image formation setup consisting of the test scene, imaging lens, and monochrome camera. The lens and camera positions are adjusted until a sharp image is formed on the sensor plane.

**Note: Spectral ground truth**

**Mission:** Acquire a hyperspectral reference dataset using monochromatic illumination.
**Deliverable:** A 31-band reference cube and a capture log with wavelength, exposure time, and gain when applicable.

## 3.3 Hyperspectral Ground Truth Acquisition

After the image formation system is aligned, the hyperspectral ground truth is acquired using the monochromator. In this stage, the scene is illuminated sequentially at different wavelengths. For each wavelength $\lambda_\ell$, one image $I(x, y, \lambda_\ell)$ is captured. After completing the scan, the set of images forms a reference hyperspectral cube with $L = 31$ bands.

For each band, the exposure time and gain should be adjusted only to avoid saturation and to maintain an adequate signal level. The acquisition parameters must be recorded because they are needed later for radiometric normalization. A consistent naming convention is recommended, for example `GT_lambda_001.tif`, `GT_lambda_002.tif`, ..., `GT_lambda_031.tif`.

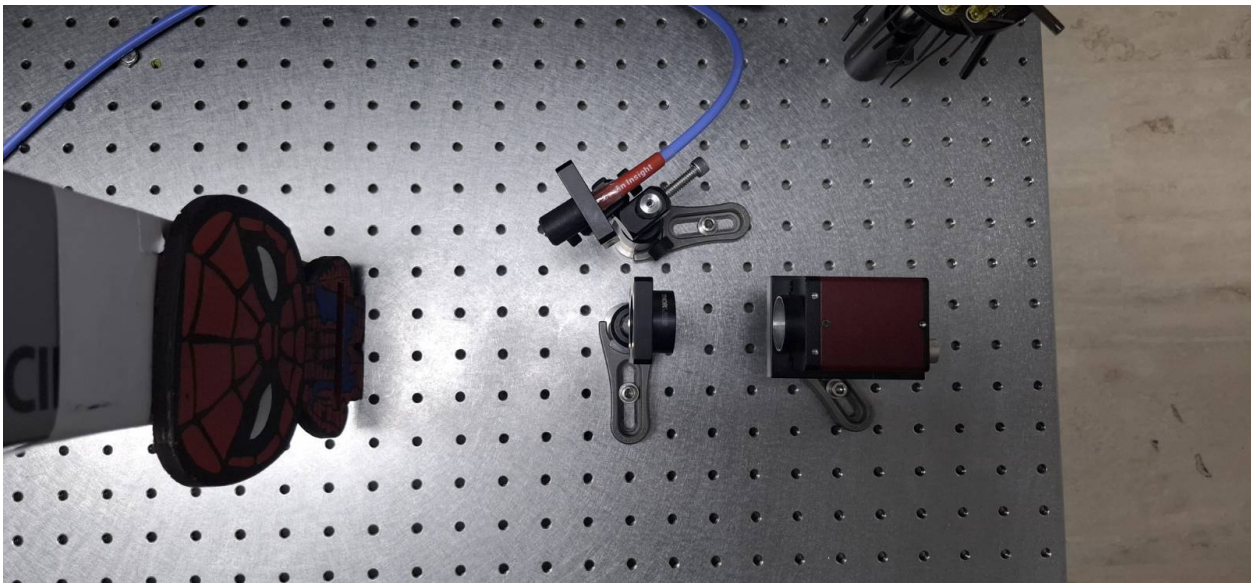


**Figure 11:** Hyperspectral ground-truth acquisition using monochromatic illumination. Each wavelength is acquired independently to construct the 31-band reference cube.

**Note: Filter-based acquisition**

**Mission:** Acquire reduced spectral measurements using discrete filters under broadband illumination.
**Deliverable:** A set of $N$ filtered images and a capture log with filter index, exposure time, gain, and wavelength information if available.

## 3.4 Multispectral Acquisition Using Spectral Filters

After acquiring the hyperspectral reference cube, the monochromator illumination is replaced by a broadband lamp. The scene, lens, and camera should remain in the same geometry as much as possible. Then, discrete spectral filters are placed in the optical path, and one image is captured for each filter.

Each filtered image represents a spectral measurement of the scene. Unlike the monochromator acquisition, a filter image is not a narrow pure band. It is a weighted measurement determined by the filter transmission, the lamp spectrum, and the camera response. If $N$ filters are used, the result is a measurement set $\mathbf{i}(x, y) \in \mathbb{R}^N$.

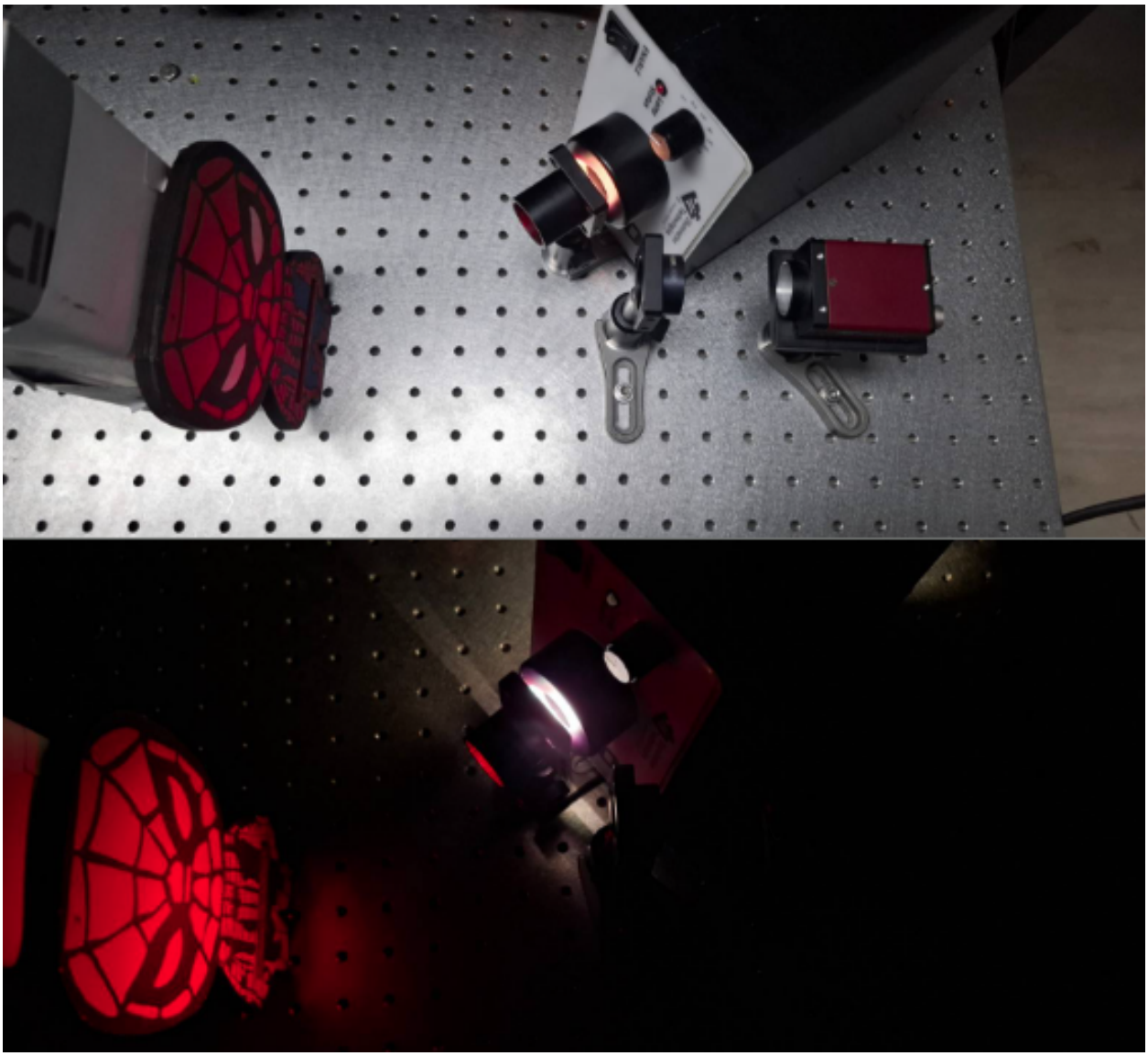


**Figure 12:** Multispectral acquisition using discrete optical filters under broadband illumination. The figure shows the illumination path, filter placement, and scene response for a selected filter. Each filter produces one measurement associated with a different spectral response.

**Note: Data preparation**

**Mission:** Organize, normalize, and align the acquired data before spectral reconstruction.
**Deliverable:** A short description of the preprocessing steps and the final data organization.

### 3.5 Data Preparation for Spectral Reconstruction

At the end of the acquisition stage, two datasets are available: the hyperspectral reference cube from the monochromator and the reduced set of filter-based images. These datasets must be prepared before reconstruction.

First, the images are organized using their acquisition order and metadata. Then, a basic radiometric normalization is applied using exposure time and gain when available. If dark frames are available, they can be subtracted to reduce the sensor offset. Next, the bands should be spatially aligned, especially if the filters or illumination changes introduced small shifts in the image. Finally, a common region of interest is cropped from both datasets.

The final output of this stage is a ground-truth cube $\mathbf{s}(x, y) \in \mathbb{R}^{31}$ and a filter-based measurement vector $\mathbf{i}(x, y) \in \mathbb{R}^{N}$. These two elements are the input for the spectral reconstruction stage.

## 4 Deliverables

**Table 3:** Deliverables (minimum required content)

| **Deliverable** | Minimum content |
|---|---|
| **Report (PDF)** | 2–4 pages including: (i) 3–5 key figures (optical setup, monochromator GT bands, filter acquisition setup, GT vs reconstructed spectrum), (ii) one small metrics table, and (iii) short answers to the analysis questions. |
| **Results folder** | A `results/` folder with exported figures (recommended) and a `README.txt` describing data naming, acquisition parameters, and the execution order. |

## 5 Analysis questions

1. **Thin lens model:** Using the thin lens equation, explain how changing the focal length affects field of view and magnification. What did you observe experimentally?

2. **Optical setup alignment:** How did you align the scene, lens, and camera? Discuss focus, field of view, vignetting, and practical constraints of the optical breadboard.

3. **Spectral imaging model:** In the integral model

$$I(x, y) = \int_{\lambda_{\min}}^{\lambda_{\max}} S(x, y, \lambda)\, E(\lambda)\, Q(\lambda)\, d\lambda,$$

explain what changes when using a monochromator versus a broadband lamp with filters.

4. **Filter measurements as projections:** In the discrete model $\mathbf{i} = \mathbf{A}\mathbf{s}$, what does each row of $\mathbf{A}$ represent physically? Why is $\mathbf{i}$ not a set of "pure bands"?

5. **Underdetermined reconstruction:** Since $N < L$, why is spectral reconstruction an inverse problem? What role does regularization play in stability?

6. **Registration and normalization:** Which step affected your results the most: radiometric normalization or geometric registration? Provide evidence (a figure or a metric change).

7. **Reconstruction performance:** Where does the reconstruction fail (which wavelength regions or scene materials)? Suggest at least one reason (filter coverage, noise, illumination, conditioning).

8. **Design question:** If you could add only 3 more filters, where would you place them (which spectral regions) and why?